Universality classes of driven lattice gases

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Motivated by some recent criticisms to our alternative Langevin equation for driven lattice gases (DLG) under an infinitely large driving field, we revisit the derivation of such an equation, and test its validity. As a result, an additional term, coming from a careful consideration of entropic contributions, is added to the equation. This term heals all the recently reported generic infrared singularities. The emerging equation is then identical to that describing randomly driven diffusive systems. This fact confirms our claim that the infinite driving limit is singular, and that the main relevant ingredient determining the critical behavior of the DLG in this limit is the anisotropy and not the presence of a current. Different aspects of our picture are discussed, and it is concluded that it constitutes a very plausible scenario to rationalize the critical behavior of the DLG and variants of it.

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The driven lattice gas (DLG) [1–3] is a simple nontrivial extension of the kinetic Ising model, and constitutes certainly a main archetype of out-of-equilibrium system. Fully understanding the critical properties of the DLG would be a fundamental milestone on the way to rationalizing the fast developing field of nonequilibrium phase transitions. The DLG is defined as a half filled, d-dimensional kinetic Ising model with conserved dynamics, in which transitions in the direction (against the direction) of an external field, E, are favored (unfavored) [1–3]. The external field induces two main nonequilibrium effects: the presence of a net current of particles in its direction, and anisotropic system configurations. At high temperatures the system is in a disordered phase, while below a certain critical point it orders by segregating into high and low density aligned-with-the-field stripes.

In order to analyze the DLG critical nature, and determine its degree of universality, a Langevin equation intended to capture the relevant physics at criticality was proposed and renormalized more than a decade ago [4]. This elegant theory, the *driven diffusive system* (DDS) seems to capture the main symmetries and conservation laws of the discrete DLG (including a current term as the most relevant nonlinearity), and is therefore a suitable and very reasonable candidate to be *the* canonical continuous model, representative of the DLG universality class.

Unfortunately, the most emblematic prediction coming from the analysis of the DDS equation, namely, the mean field behavior of the order parameter critical exponent ($\beta=1/2$ [4]), has not been compellingly verified in any Monte Carlo simulation of the DLG in spite of the huge computational effort devoted to test it. In particular, systematic deviations from scaling are observed both in $d=2$ [3,5] and in $d=3$ [6] if data collapse is attempted using $\beta=1/2$ [7]. On the other hand, different Monte Carlo numerical simulations (performed in different geometries and using different finite size scaling ansatzs) lead systematically to a value of β around 0.3 with error bars apparently excluding $\beta=1/2$ (we refer the interested reader to [3] for a review of simulation analysis). This is a main indication that the DDS Langevin equation does not describe properly the DLG at criticality.

Moreover, there are some other hints suggesting strongly that the discrepancies between the predictions of the standard theory and Monte Carlo results are more fundamental than a simple numerical difference in β. In particular, the intuition developed from Monte Carlo simulations of the DLG and variants of it [3,8] (performed under large external driving fields) suggests that, contrarily to what happens for the DDS equation, it is the anisotropy and not the presence of a current the most relevant ingredient for criticality. For instance, in a modification of the DLG in which anisotropy is included by means other than a current [9], the scaling behavior at criticality remains unaltered upon the switching on of an infinite driving (see the Appendix and [9,3]). Other compelling evidences supporting this hypothesis can be found in [3,10].

In order to shed some light on this puzzling situation and reconcile theory with numerics, different possible scenarios have been explored; but so far, no satisfactory clarification has been reached. Within this context, we have recently revisited the time-honored DDS equation and questioned its general validity [11,12]. In particular, we have tackled the task of constructing a coarse-grained procedure in a more detailed way such that, starting from a master equation representing the DLG, would give as output a continuous Langevin equation. This approach permits us to keep track of microscopic details that could eventually be overlooked when writing down a Langevin equation respecting naively the microscopic symmetries and conservation constraints [4]. This approach has given rise to a rather unexpected and quite interesting output: The limit of infinitely large driving (i.e., the limit in which attempted jumps in the direction of the field are performed with probability one and jumps against E are strictly forbidden) is singular [11]. Let us stress that in order to enhance nonequilibrium effects most of the available computer studies are performed in this limit. The main results derived so far using our approach are

(i) For vanishing values of the driving field it leads to the standard equilibrium model *B* [13], capturing the relevant physics of the kinetic Ising model with conserved dynamics.

(ii) For nonvanishing, but *finite* driving fields we reproduce the standard DDS Langevin equation [2,4].

(iii) In the limit of infinitely large driving, where the dependence of jumps in the direction of the field on energetics is replaced by a zero-one (all-or-nothing) condition, a different Langevin equation emerges. This new equation has the main property of not including any relevant term coupling E to the density field ϕ [11], and the presence of anisotropy is its main relevant ingredient.

The new Langevin equation for the DLG under infinitely large driving proposed in [11,12] and renormalized in [14] has some important virtues to be discussed afterwards, but seems also to exhibit some pathologies, as recently pointed out by Caracciolo *et al.* [15] and also by Schmittmann *et al.* [16]. In what follows we show that such anomalies can be healed in a rather natural way, and do not disprove at all the general validity of our new approach (as could be inferred from [16]).

Let us now present the Langevin equation derived in [11,12] for the infinite driving limit, report on its deficiencies [15,16], and discuss the way to heal them. The equation reads [11,12]

$$\partial_t\phi=\frac{e_0}{2}\left[-\Delta_\parallel\Delta_\perp\phi-\Delta_\perp^2\phi+\tau\Delta_\perp\phi+\frac{g}{3!}\Delta_\perp\phi^3\right]+\sqrt{e_0}\boldsymbol{\nabla}_\perp\cdot\boldsymbol{\xi}_\perp+\sqrt{\frac{e_0}{2}}\nabla_\parallel\xi_\parallel, \tag{1}$$

where $\nabla_\parallel$ ($\nabla_\perp$) is the gradient operator in the direction parallel (perpendicular) to the driving field, and ξ is a conserved Gaussian white noise [11,14]. This equation is analogous to a model *B* in the direction(s) perpendicular to the field, coupled to a simple random diffusion mechanism in the parallel direction. The origin of all the difficulties pointed out in [15,16] can be traced back to the following property: Defining the total density for each value of $r_\parallel$, $\Upsilon(r_\parallel,t)\equiv\int d^{d-1}r_\perp\phi(r_\parallel,\mathbf{r}_\perp)$, it is not difficult to see (after averaging over the noise) that $\Upsilon(r_\parallel)$ is a conserved quantity for all values of $r_\parallel$ [15]. Observe also that $\Upsilon(r_\parallel)$ is nothing but the zero Fourier mode of the density at each column. These (spurious) conservation laws, absent in the DLG, are at the origin of the infrared singularities appearing in Eq. (1) [15,16].

In order to investigate the causes of this deficiency in our Langevin equation and eventually overcome the problem of the extra conservation laws and associated infrared divergences, we have re-analyzed our derivation of Eq. (1) in [11,12]. One can easily see that the transition rates in the microscopic master equation in [11,12] were written as depending on the variations of two adding contributions: the free energy functional (the usual Ginzsburg-Landau free energy) and the external driving-field contribution. The transition rates, written in that way, saturate to zero or one in the field direction, in the limit of infinite driving. This saturation erases any further dependence on the free energy density (which includes both entropic and energetic contributions). On the contrary, in the DLG it is only the dependence on the Ising energetics that becomes negligible in the limit of large driving fields. In a coarse-grained description we should therefore separate energetic from entropic terms. With this guiding idea, we have reconsidered our derivation of Eq. (1) and rewritten the transition rates in [11,12] as the product of two contributions: one controlling the energetics and the other one the entropic part [17]. By performing a calculation analogous to that in [11,12], but including the transition rates written in this modified way, it is a matter of algebra to see that a new term (missing in [11,12,14]) emerges: $\rho\nabla_\parallel\phi(\mathbf{x},t)$ [18]. It is straightforward to verify that apart from properly keeping track of entropic contributions, this extra (mass) term heals all of the aforementioned problems in Eq. (1): no spurious conservation laws are involved and generic infrared singularities disappear.

Let us now discuss how this new additional term affects the results presented in [14]. Performing a naive scaling analysis, one sees that $x_\parallel\sim x_\perp^2$ [19], and upon elimination of naively irrelevant terms and absorbing e_0 into the time scale, one obtains our final result: the critical Langevin theory under infinitely large driving,

$$\partial_t\phi=\rho\Delta_\parallel\phi-\Delta_\perp^2\phi+\tau\Delta_\perp\phi+\frac{g}{3!}\Delta_\perp\phi^3+\frac{2}{\sqrt{e_0}}\boldsymbol{\nabla}_\perp\cdot\boldsymbol{\xi}_\perp, \tag{2}$$

that we call the *anisotropic diffusive system* (ADS). This turns out to be a well known Langevin equation: the continuous representation of the randomly driven DLG [20,2], i.e., a DLG in which the external field changes sign randomly in an unbiased fashion. The main difference between this theory and the DDS is that the ADS does not include an overall current. The current term $E\nabla_\parallel\phi^2$ appearing in the DDS (and constituting its most relevant nonlinearity) is absent here. In the random DLG such a term cannot appear for symmetry reasons, while in the infinite driving case discussed in this paper it is the saturation of the transition rates in the field direction that prevents such a current term from appearing.

The cubic operator and the Laplacian term in the parallel direction in Eq. (2) are both marginal at the critical dimension $d=3$. The results up to first order in an ε expansion of Eq. (2) around $d=3$ are [20,21]: $\nu_\perp=1/2+\varepsilon/12$, and $\beta=1/2-\varepsilon/6$ [21]. Observe that in $d=2$ one obtains $\beta=1/3$ (slightly modified by two-loop corrections [20]), in remarkable good agreement with Monte Carlo results. For instance (see Table I and [23]): the best available Monte Carlo result for the random DLG is $\beta\approx0.33$ [20]; for the infinitely driven DLG $\beta\approx0.30\pm0.05$ [3]; and $\beta\approx0.34$ for the closely related model studied in [9], called anisotropic lattice gas automation (ALGA), and argued to belong to the same universality class (see the Appendix).

Some further comments on the validity of our approach follow: (i) Our complete theory (including constant and irrelevant terms) does have a net current [11,12], though it does not enter the final Langevin equation. (ii) An infinitely large field is, in practice, any for which transitions against the field never occur. Given that all commonly used transition rates depend on E through exponential functions, field values much larger that unity can be considered infinite for all practical purposes in Monte Carlo experiments. For

TABLE I. Universality classes for different models as a function of the driving field value. The reported values of β correspond to the best-to-date numerical results in $d=2$. The theoretically predicted value is $\beta=1/2$ for the DDS and $\beta\approx0.33$ for the ADS.

Model	No net current	Net current Finite driving	Net current Infinite driv.
DLG	Model *B* $\beta=0.125$	DDS	ADS $\beta\approx0.3$
Random DLG	ADS $\beta\approx0.33$	DDS	ADS
ALGA	ADS $\beta\approx0.34$	Undefined	ADS $\beta\approx0.34$

smaller fields, we expect crossover effects from the infinite field regime (ruled by the ADS) to the finite-driving standard DDS behavior to occur. These crossovers could obscure the numerical observation of the DDS mean-field exponent beta for large but finite driving fields. (iii) The introduction of the new term in the parallel direction heals all the possible problems in relation to infrared singularities, extra conservation laws, and anomalies in the structure function [16]. In particular, the structure function presents a discontinuity singularity as happens in the DLG [2]. (iv) Given the absence of any relevant current term in Eq. (2), the critical theory has ''up-down'' symmetry ($\phi\rightarrow-\phi$). This symmetry, in principle, is absent in the microscopic model, as the presence of nonvanishing three-point correlation functions seems to indicate [2]. However, it is not clear whether such correlations are relevant at criticality or not. As an indication that in fact they could well be irrelevant, we discuss here the problem of triangular anisotropies [22]: In both the DLG and the DDS, droplets of the minority phase (if any) develop triangular shapes, closely related to the existence of nonvanishing three-point correlations. However, triangles orientate in opposite directions in the microscopic DLG and in the continuous DDS. This difference seems to be not universal, as shown by recent Monte Carlo studies [22], i.e., it depends on microscopic details and can be modified by changing them both in the DLG and in the DDS. This fact supports the idea that nonvanishing three-point correlation functions might not be a relevant ingredient for a description of the DLG at criticality. More significatively, simulations show that *for large enough driving fields, the triangular anisotropy is suppressed* (see [22]), providing an indication that the up-down symmetry is restored in the infinite driving limit. This constitutes, we believe, another strong backing of our picture.

In summary, we have discussed the plausibility of the alternative field-theoretical approaches to driven lattice gases under the effect of an infinitely large external driving field. Some deficiencies recently pointed out are overcome by introducing an extra Laplacian term in the direction of the field in the Langevin equation first proposed in [11,12]. This new term, coming from a proper consideration of entropic contributions, had been overlooked in previous papers. Our approach leads to the following global picture: (i) For $E=0$, model *B* reproduces the equilibrium critical properties of isotropic diffusive systems. (ii) For finite driving field, the standard DDS Langevin equation, including a current term, should describe properly the long wavelength properties around the critical point. (iii) The limit of infinitely fast driving is singular: *the current is irrelevant, and the anisotropy becomes the main relevant property*. The leading critical properties in this case are expected to be described by the ADS, Eq. (2). The reason for this being that in the presence of infinite driving the transition rates saturate to 1 (0) for allowed (forbidden) transitions in the driving direction, and no further track of coupling between E and the density field survives in the resulting Langevin equation. We want to stress at this point that this property was not obvious *a priori*, but emerges as a natural output from our model building strategy.

The proposed Langevin equation for large external field provides a quite plausible scenario shedding some light on a difficult problem. In particular, it justifies the observed lack of differences (for large fields) in simulations in systems with and without a current, and provides a likely justification of why the standard prediction $\beta=1/2$ is not confirmed in Monte Carlo simulations, and instead a value $\beta\approx0.33$ is observed.

In order to test numerically the picture presented in this paper, it would be highly desirable to perform extensive simulations for finite driving field ($E\approx1$), and study whether differences with respect to the available Monte Carlo results for large fields appear. It would also be interesting to improve the finite size scaling analysis following the strategy used in [24,15].

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APPENDIX

As an evidence aimed to transmitting the intuition that, for infinitely large driving, the current is not relevant at criticality, let us briefly discuss in this appendix a rather compelling Monte-Carlo observation. It corresponds to a variation of the DLG, named ALGA; see [9] for a detailed definition. This model is placed by definition at the limit of infinite driving: jumps in the anisotropy direction are performed randomly without attending to energetic considerations. Simulations are performed both in the presence of an overall current (case $p\neq1/2$ in [9]) and in the absence of it ($p=1/2$); the curves for the order parameter versus the distance to the critical point are indistinguishable in the cases with and without a current (Fig. 3 in [9] is particularly illuminating). It could be argued that the details of this modified model [9] render it not completely equivalent to the original DLG. However, we do not think these microscopic differences have any relevance at a coarse-grained level. In fact, we expect this model to be represented by Eq. (2): in one direction particles tend to stay together, and it is natural to assume that their coarse-grained behavior is controlled by a model *B* in this direction. In the other direction, jumps occur regardless of energetics and, therefore, the dynamics becomes purely diffusive. With these two ingredients we recover the ADS, Eq. (2), as the Langevin equation for the ALGA. As a further evidence supporting this hypothesis let us mention that the measured β exponent in the ALGA is $\beta \approx 0.34$ (again very close to the value $\beta \approx 1/3$) in both cases: with and without current.

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